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M.Sc. Sem-II, Paper VI [MAT CC-06]

Complex Analysis :- Algebra of Complex number

As we know $x^2+1=0 \Rightarrow x^2=-1$ has no real solutions, naturally, we want its roots. So we show a new symbol for the roots and call it a complex number.

Def:- The symbols $\pm i$ will stand for the solutions of the eqn. $x^2=-1$, we will call these new numbers as complex numbers. written as

$$\sqrt{-1} = \pm i.$$

The number i is called an imaginary number. These are perfectly valid numbers that don't happen to lie on the real number line.
We are going to look at the algebra, geometry and most important exponentiation of Complex numbers.

ex. Solve the eqn $x^2+x+1=0$.

Soln- By quadratic formula, here

$$x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}.\sqrt{-1}}{2}$$

$$= \frac{-1 \pm \sqrt{3}\, i}{2} = \frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}$$

A polynomial of degree n has exactly n complex roots, where repeated roots are counted with multplicity. This is the fundamental theorem of algebra.

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- Complex numbers are defined as the set of all numbers $z = x + yi$ or $z = x + iy$, where x and y are real.
- We denote the set of all complex numbers by $\mathbb{C}$.
- We call x the real part of z. This is denoted by $x = Re(z)$.
- We call y the imaginary part of z. This is denoted by $y = Im(z)$.

Note:- The imaginary part of z is a real number. It does not include the i.

The basic arithmetic operations follows the standard rules. We know that $i^2 = -1$. We will go through these using some simple examples.

1. Addition :- $(3+4i) + (7+5i) = 10 + 9i$
2. Subtraction:- $(3+4i) - (7+5i) = -4 - 1i$
3. Multiplication:- $(3+4i)(7+5i) = 21 + 15i + 28i + 20i^2$

$$= 21 + 43i - 20 = (21-20) + 43i$$

$$= 1 + 43i \quad [\because i^2 = -1]$$

Before consider about division and absolute value we introduce a new operation called conjugation. It will prove useful to have a name and symbol for this.

Complex conjugation is denoted with a bar and defined by $\overline{x+iy} = x - iy$.

If $z = x + iy$ then its conjugate is $\bar{z} = x - iy$ and we read this as "z-bar $= x - iy$".

Ex. $\overline{3+5i} = 3 - 5i$.